Grade 7

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 3

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the ‘District Development Programme’. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let’s work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

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Contents

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of "saying it, showing it and symbolising it". This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "say it" or auditory mode.
- The pictures and diagrams provide suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

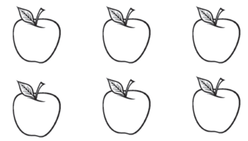
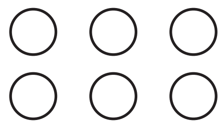
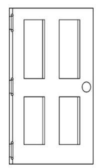
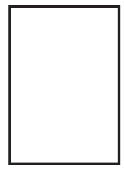
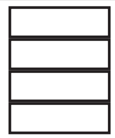
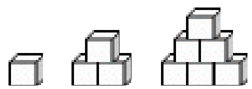
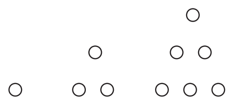
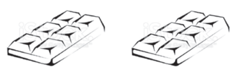
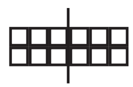
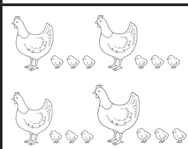
Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

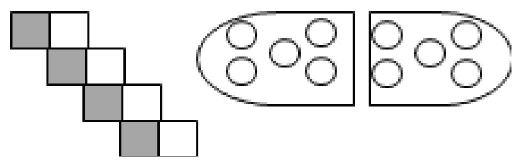
- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

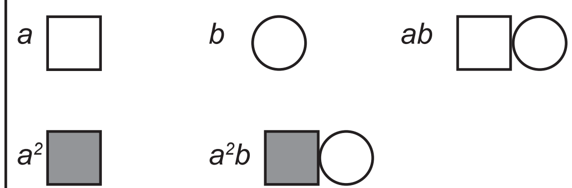
CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	**Real or physical For example:**	**Picture**	**Diagram**	**Number (with or without unit)**	**Calculation or operation, general form, rule, formulae**
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160 = 550cm$ Area: $L \times W = 195 \times 80 = 15\,600cm^2 = 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml = 1$ litre or 1 litre $\div\ 4 = 0.25$ litre
Patterns	Building blocks			1; 3; 6...	$\frac{n(n+1)}{2}$
Fraction	Chocolate bar			6 12	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of $12 = 6$
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	$500g = 0{,}5$ kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept	SHOW IT: to embody the idea	SYMBOL IT: to enable mathematising
	• Essential for introducing terminology in context • Supports learning through the auditory pathway • Important to link mathematics to everyday realities	• Essential to assist storing and retrieving concepts • Supports understanding through the visual pathway • Important to condense a variety of information into a single image	• Essential to assist mathematical thinking about concepts • Supports the transition from situations to mathematics • Important to expedite calculation and problem solving
FP: Doubling and halving	"To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use 2½ cups of flour, it means she has to use twice as much this time." "To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro's in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half."	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example: Double  Halve  3. Diagrams: 	$7 + 7 = \square$ $7 + \square = 14$ $\triangle + \bigcirc = 14$, but $\square + \square = 14$ 2 times 7 = 14 double of 7 is 14 $14 - 7 =$ $14 - \square = 7$ 14 divided by 2 = 7 14 halved is 7

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?**"**	o o oo o oo ooo Draw the next term in this pattern. T1 T2 T3 T4 o o oo o oo ooo o oo ooo oooo Describe this pattern. What is the value of the 9th term of this pattern (T9)? o o oo o oo ooo o oo ooo oooo o oo ooo oooo ooooo To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.

SP: Grouping the terms of an algebraic expression	*"We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:"*	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagrammatic form 	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: NUMERIC AND GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area, ‘Patterns, Functions and Algebra’ and counts for 25 % in the final exam.
- The unit covers identification, description and extension of a variety of number patterns and is an essential element required in the clear understanding of Algebraic conceptual knowledge.
- It is important to ensure that learners can perform all the necessary operations such as determining the next terms of a sequence or determining the rule that gives the required sequence. These basic concepts are the required foundations for a concise understanding of Algebraic concepts and Functions later in the FET phase.
- Although learners have been working with numbers and their various properties as well as with simpler versions of number patterns it is important that learners start to expand their understanding of the formulae or rules that form the relationships within a sequence.
- Practice as much as possible – if the textbook used does not have enough exercises, make an effort to find some more or make up a worksheet of your own.
- In Term 4 patterns will be covered again, where they could include integers. In this term patterns must be restirced to whole numbers, numbers in exponential form, common fractions and decimals.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE	GRADE 7	GRADE 8 &9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Using flow diagrams to demonstrate inverse operations, associative property and multiplication strategies.
• Multiple operations performed using flow diagrams.
• Extending geometric patterns or sequences with a constant difference or ratio
• Determining the rule of a geometric pattern
• Representing patterns in a variety of ways
	• Investigate and extend numeric and geometirc patterns looking for relationships between numbers, including patterns:
	  • represented in physical diagram form
	  • not limited to sequences involving constant difference or ratio
	  • of learners own creation
	  • represented in tables
	• Describe and justify the general rules for observed relationships between numbers in own words
		• Investigate and extend patterns (numeric, geometirc) represented visually or in tables
		• Algebraic representation of patterns
		• Describe and justify the general rules in own words and in algebraic language

GLOSSARY OF TERMS

Term	Explanation / Diagram
Pattern	A series or sequence that is repeated, according to a rule
Number Patterns	A sequence of numbers that are ordered based on a rule
Rules	The explanation of how the pattern is arranged
Term	A number or the combination of a number and a variable in a numerical pattern or mathematical expression.
Constant Difference	A pattern has a constant difference if the same number can be added or substracted to get from one term to the next term
Constant Ratio	The number used to multiply one term to get to the next term in a geometric sequence. If division occurs, reciprocate and turn into multiplication. For example $\div 5 = \times \frac{1}{5}$
Sequence	Numbers that follow each other in order
Geometric Patterns	These are patterns that are represented by diagrams. Each diagram is altered to the next by applying a constant rule.

SUMMARY OF KEY CONCEPTS

Recognising and extending a numeric pattern:

Teaching Tip:
Learners have been working with this concept throughout the intermediate phase. They have continuously described patterns and extended them by using their own language and not bound by any conventional methods. In Grade 7, the idea is to start at a similar level and then extend the learners to use conventional, algebraic language and methods to perform the same task.

1. Learners need to be making use of correct terminology such as constant difference and constant ratio when describing the patterns and must refer to each portion of the pattern as a term.
2. For example: 1; 3; 9; 27 has a constant ratio of 3. (This is the correct language required at Grade 7 level instead of " you keep 'timesing' by 3")
3. Learners need to be able to extend a numeric pattern as well as determine a missing term within a numeric sequence.

Teaching Tip:
Sequential teaching of this section is important. It is therefore an excellent idea to start with patterns that have a constant difference and then progress to patterns with a constant ratio.

For Example:
Describe the following pattern (by giving a rule) then extend the pattern by writing the next three numbers.

1. 2 ; 7 ; 12 ……
2. 120 ; 113 ; 106 …..
3. 2 ; 6 ; 18 …..
4. 500 ; 250 ; 50 …

Solutions:
1. 5 is added to one term to get the next term (there is a constant difference of 5)
   - Rule: +5
   - Next three terms: 17 ; 22 ; 27

2. 7 is subtracted from one term to get the next term (there is a constant difference of minus 7)
   - Rule: -7
   - Next three terms: 99 ; 92 ; 85
3. One term is multiplied by 3 to get the next term (there is a constant ratio of 3)
   - Rule: x 3
   - Next three terms: 54 ; 162 ; 486
4. One term is divided by 5 to get the next term (there is a constant ratio of $\frac{1}{5}$)
   - Rule: $\times\frac{1}{5}$
   - Next three terms: 10 ; 5 ; $\frac{5}{2}$ (or $2\frac{1}{2}$)

Although the concept of patterns has been covered previously it is wise to use pictures and concrete examples as patterns are explained.

Investigating and extending numeric patterns with neither constant differences nor ratios

This is a new extension to the numeric patterns that learners are exposed to as they will no longer be able to rely on a constant change between the terms in the sequence.

Use this opportunity to remind learners of when they used complex flow charts in grade 6 and the start of the year where more than one operation was performed on the input to result in the output.

For example:
Describe the rule for this number pattern and give the next three terms of the pattern:

8; 10; 14; 20; 28

The pattern above increases by multiples of 2 so to progress from the first term we would add 2 to get to the next term but then we would add 4 to get to the third term. The next three terms would be 38 (+10); 50 (+12); 64 (+14).

Teaching tip:
Once learners have had the opportunity to practise these types of patterns on their own, consolidate their learning by allowing them to work in pairs. Each learner creates their own pattern and their partner needs to explain their pattern back to them with the rule and by extending the pattern further. Once the activity is complete, show learners the Fibonacci sequence and let learners discuss/offer their opinion on the rule and an extension of the pattern.

Fibonacci sequence: 1 ; 1 ; 2 ; 3 ; 5 ; 8 ….

(From the third term onwards, the previous two terms are added to get the next term. 1 + 1 = 2; 1 + 2 = 3; 2 + 3 = 5 etc)

Using tables to investigate and extend numeric patterns

1. Using tables can be very useful when learners need to identify and extend numeric patterns. Learners need to be encouraged to determine the rule for patterns. In other words learners may not always be required to find the next three terms. They may be asked for the 10th or 20th term
2. For example: Find the 20th term (the term in position 20) of the following pattern:

Position in sequence	1	2	3	4	20
Term	3	6	9	12	?

The sequence is made up of the multiples of 3 therefore the 20th term is 20 multiplied by 3 which would be 60.

Find the rule to describe linear patterns

Linear patterns have a constant difference. This constant difference is essential to finding the rule of a pattern.

For example:
Consider the pattern 4 ; 7 ; 10 ….
The common difference is +3

Consider the pattern 12 ; 7 ; 2….
The common difference is -5

Steps to follow to find the rule of a linear pattern:

- Find the constant difference
- Using the idea that you know what the first term is, multiply the constant difference by 1 (representing the first term) and find what still needs to be done to get the first term
- Check the rule works for term 2 and term 3.

For example:
Find the rule for the pattern: 9 ; 14 ; 19 ; 24…..

This is easier if represented in table form:

1	2	3	4	5
9	14	19		

Common difference is +5

$T_1 = 5(1) + \mathbf{4}$ (this will give the 9, the **1st** term)
$T_2 = 5(2) + \mathbf{4}$ (this will give the 14, the **2nd** term)
$T_3 = 5(3) + \mathbf{4}$ (this will give the 19, the **3rd** term)

$\therefore$ the rule is to multiply by 5 and add 4.
(In words: ‘5 x the position of the term + 4)

This can be written as $T_n = 5n + 4$ where ‘n’ can represent any position in the pattern.

The rule can now be used to find a term in any position.

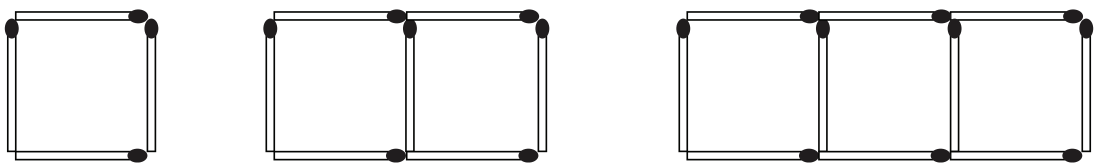
For example:
Find the 15th term in the above pattern.

Solution: $T_{15} = 5(15) + 4 = 79$

Geometric Patterns

1. Geometric Patterns are numeric patterns represented diagrammatically. The diagram represents the structure of the number pattern so learners need to be able to relate the visual (diagram) to the abstract (rule)
2. Learners need to be able to draw the next diagram in the sequence as well as be able to represent the diagram as a numeric sequence.
3. For example:

Look at the pattern below that represents squares made with matchsticks. Count the number of matchsticks used to make each pattern.



This information would be easier read in a table:

1	2	3	4	5
4	7	10		

The first pattern (1) uses 4 matches

The second pattern (2) uses 7 matches ….

Learners should recognise that this is a linear pattern with a common difference of 3. This means they could find the rule and be able to say how many matches would be used for the any pattern (term) further down the sequence.

Teaching Tip:
Learners can take some to time visually represent patterns using matches or small sweets (jelly tots). Remember that we are moving to the abstract, but should a learner be struggling to see these relations then backtracking to the visual will help them see the pattern. Practice will help learners see the patterns develop. Keeping these lessons practical will also assist learners in their understanding of the concepts.

TOPIC 2: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit covers identification, description and extension of a variety of number patterns and is an essential element required in the clear understanding of algebraic conceptual knowledge.
- It is important to ensure that learners can perform all the necessary operations such as determining inputs, outputs and rules. These basic concepts are the required foundations for a concise understanding of algebraic concepts and functions later in the FET phase.
- Although learners have been working with basic flow diagrams and tables of values, it is important that learners start to expand their understanding of the formulae or rules that form the relationships between input and output values.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/ GRADE 6	GRADE 7	GRADE 8 and 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Use of flow diagrams to demonstrate inverse operations, associative property and multiplication strategies.
• Introduction to finding the input, output and rule of number sequences
	Input and output values
	• Determine input values, output values or rules for patterns and relationships using:
	  • flow diagrams
	  • tables
	  • formulae
	Equivalent forms
	• Determine, interpret and justify equivalence of different descriptions of the same relationship or rules presented:
	  • Verbally
	  • in flow diagrams
	  • in tables
	  • by formulae
	  • by number sentences
		Input and output values
		• Determine input values, output values or rules for patterns and relationships using:
		  • flow diagrams
		  • tables
		  • formulae
		  • equations
		Equivalent forms
		• Determine, interpret and justify equivalence of different descriptions of the same relationship or rules presented:
		  • Verbally
		  • in flow diagrams
		  • in tables
		  • by formulae
		  • by equations
		  • by graphs ona Cartesian plane

GLOSSARY OF TERMS

Term	Explanation / Diagram
Input	The starting value or the independent value in a flow diagram
Output	The final value or the dependent value in the flow diagram. It is the value obtained after the rule has been applied to the input value.
Process or Rule	Process or Rule is what is done to the input value that results in the output value. These can be a single operation or a variety of operations applied to the input value.
Flow Diagram	A diagram that shows the input, process and the result or output value diagrammatically. It is a diagrammatical representation of a number sentence.

SUMMARY OF KEY CONCEPTS

A function is a special rule or relationship between values

Inputs and Outputs

Every value you put into a function (input) has a specific value that comes out (output) after one or more operations have been performed.

Flow Diagrams

Flow diagrams show how input numbers are changed to become output numbers.
Mathematical rules are used to show what operations have been applied to the input numbers in order to get the output numbers.
A flow diagram is similar to an equation, just written differently.
Example:

Equation	Flow Diagram
$3 + 2 = 5$	$3 \rightarrow +2 \rightarrow 5$

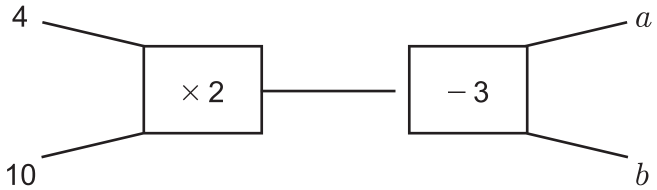
Flow diagrams can be changed into equations when solving is required.

Flow Diagram	Equation
$a \rightarrow +2 \rightarrow = 7$	$a + 2 = 7$

Patterns can be represented algebraically. This means that variables (letters that can represent many values) will be used.

Finding output when given the input and the rule

We can use flow diagrams to find missing values.
For example:

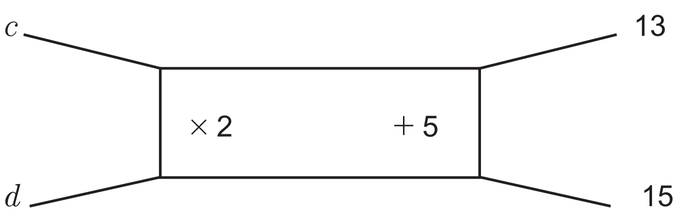


To find 'a': Start with 4, multiply by 2 and then subtract 3
(a = 5)
To find 'b': Start with 10, multiply by 2 and then subtract 3
(b = 17)

Finding input when given the output and the rule

Since we are working backwards, we need to work with INVERSE operations in order to 'undo' the expression.

For example:



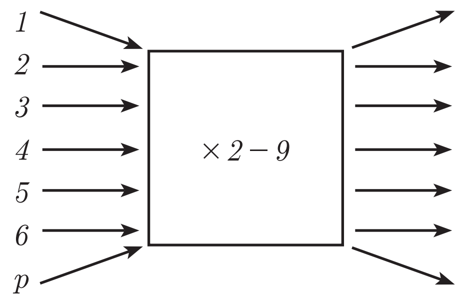
To find 'c': Using inverse operations:
Start with 13, <u>subtract</u> 5 (inverse operation to addition)
then <u>divide</u> by 2 (inverse operation to multiplication)
(c = 4)

To find 'd': using inverse operations:
Start with 15, subtract 5
then divide by 2
(d = 5)

Encourage learners to check their answers by working forwards again.

Equivalent forms

Relationships can be written in many different forms.

<table>
<tr><td>Verbally</td><td>Multiply a number by 2 and subtract 9</td></tr>
<tr><td>Flow diagrams</td><td>
</td></tr>
<tr><td>Tables</td><td><table><tr><td>input</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>output</td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr>
<tr><td>Formulae</td><td>$p \times 2 - 9 = 2p - 9$</td></tr>
<tr><td>Number sentences</td><td>$1 \times 2 - 9 = -7$
$2 \times 2 - 9 = -5$
$3 \times 2 - 9 = -3$
$4 \times 2 - 9 = -1$
$5 \times 2 - 9 = 1$
$6 \times 2 - 9 = -3$</td></tr>
</table>

Learners should recognise that the output of the above relationship forms a pattern with a common difference of 2.

TOPIC 3: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, ‘Patterns, Functions and Algebra’ and counts for 25 % in the final exam.
- The unit covers a basic introduction to variables used in mathematics as unknown values and is an essential element required in the understanding of algebraic conceptual knowledge.
- It is important to ensure that learners clearly understand mathematical language and how to replace unknown values with a variable. These basic concepts are the required foundations for a concise understanding of algebraic concepts, equations and functions later in the FET phase.
- Although learners have been working with number sentences it is important that they progress in understanding algebraic purposes in setting a general solution for a problem that may need to be solved or simplified.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 6	GRADE 7	GRADE 8 and 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Number sentences • Introduction to Algebraic Expressions • Write number sentences to describe problem situations	• Use of Mathematical language with a focus on algebraic language • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and equations.	Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and/or equations • Recognize and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coefficients an exponents in algebraic expressions

GLOSSARY OF TERMS

Term	Explanation / Diagram
Algebra	The branch of mathematics that deals with generalised arithmetic by using letters or symbols to represent numbers.
Expression	A mathematical statement which can include variables, constants and operations For example 3a + 4
Symbol	Examples include a + sign that replaces the word add.
Variable	This is a symbol (usually a letter) that may take the place of a value from a range of possible values.. For example, in the expression $x - 2$, x is the variable
Coefficient	A constant by which a variable is multiplied. For example for $5a$, 5 is the coefficient. However, a variable could be representing the coefficient. For example in ax, a is the coefficient of x
Constant	This is a value that remains unchanged. For example, in the expression $a + 5$, 5 is the constant
Term	Part of an algebraic expression. Each term is separated by a + or – sign

SUMMARY OF KEY CONCEPTS

Recognize and interpret rules or relationships represented in symbolic form

1. Learners have worked with number sentences derived from input/output diagrams and in the work covered earlier in the year as well as more recently when number patterns were covered.

   Remind learners of the work covered when they were finding the rule for number patterns at the beginning of this term. It is also helpful to start by going back to this concept and using it as a platform to introduce the idea of a variable.

2. Learners need to begin to replace the older symbolic descriptors for an unknown amount with recognised symbols or variables when given words that need to be converted into mathematical language.
3. Example: $4 + \square = 7$ (The acceptable notation used in previous grades, particularly the foundation phase) must now be converted into an acceptable algebraic general rule $4 + x = 7$.

Teaching Tip:
Remind learners that this was introduced in the numeric patterns section covered earlier in the term.
Show several examples where pictures have been used to represent unknowns and how this becomes difficult as the number of terms increases.

4. Learners need to be able to set up an expression by replacing words with acceptable symbols.
   For example:
   Half of 100 is decreased by 28 should become $\frac{100}{2} - 28$
5. Should there be an unknown value then learners need to replace this with a variable.
   For example:
   A man is 5 years older than his wife would become $x + 5$ to represent his age

Teaching Tip:
Learners may confuse < and > symbols so revise the correct use of these symbols and ensure that learners have a good understanding of the symbols that replace words such as sum, difference, 3 times as many and results in. Remind learners that < looks like an 'L' and is therefore the less than sign.

Topic 3 Algebraic Expressions

Identify variables and constants in given formulae and equations

1. Learners need to be able to identify the components of an algebraic expression. This means they must know what each component is called and how to identify that component.

Teaching Tip:
Have learners circle or underline the various components in different colours, this will highlight their purpose and make it easier for learners to identify them in future.

2. Example: $\boxed{3x - 5}$

Expression

Coefficient Variable Constant

This expression is made up of two terms.

TOPIC 4: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit covers a basic introduction to variables used in number sentences as unknown values and is an essential element required in the clear understanding of Algebraic conceptual knowledge.
- It is important to ensure that learners clearly understand mathematical language and how to replace unknown values with a variable. These basic concepts are the required foundations for a concise understanding of Algebraic concepts, equations and functions later in the FET phase.
- Although learners have been working with number sentences it is important that they progress in understanding algebraic purposes in setting a general solution for a problem that may need to be solved or simplified.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 6	GRADE 7	GRADE 8 and 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Number sentences
• Introduction to Algebraic Expressions
• Write number sentences to describe problem situations
	• Write number sentences to describe problem situations
	• Analyse and interpret number sentences that describe a given situation
	• Solve and complete number sentences by:
	  • inspection and,
	  • trial and improvement
	• Identify variables and constants in given formulae or equations
	• Determine the numerical value of an expression by substitution..
		• Solving linear equations in a variety of contexts including algebra, trigonometry, geometry and calculus.
		• Using substitution to determine values in financial mathematics
		• Setting up and solving advances equations relating to a variety of word problems
		• The solution of simultaneous equations to determine intersection of functions.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	The formal word for a number sentence. A mathematical statement including an equal sign
Variable	This is a symbol (usually a letter) that may take the place of a value from a range of possible values.. For example, in the expression $x - 2$, x is the variable
Coefficient	A constant by which a variable is multiplied. For example for $5a$, 5 is the coefficient. However, a variable could be representing the coefficient. For example in ax, a is the coefficient of x
Constant	This is a value that remains unchanged. for example, in the expression $a + 5$, 5 is the constant
Term	Part of an algebraic expression. Each term is separated by a + or – sign
Substitution	Replacing the unknown with a calculated or given amount to determine the value of the expression

SUMMARY OF KEY CONCEPTS

Writing, analysing and interpreting an equation

1. Learners have been working with number sentences derived from input/output diagrams in the work covered earlier in the year as well as at the beginning of the term when they worked with number patterns.

Teaching Tip:
Learners have been writing number sentences using input and output diagrams throughout the intermediate phase. They need to understand that they are performing the same task in a different context.

2. Learners have already been shown that they can replace unknown values with a variable. They have also written algebraic expressions and have identified the parts of these expressions.
3. Using a table to show the differences between number sentences and algebraic equations is a good method to ensure that knowledge transfer is concise.

Teaching Tip:
Tables such as the one given below are a good way to show that algebraic equations are just number sentences written in a different manner. Learners often have concerns over their own understanding of abstract ideas so it is very important that learners become more comfortable with algebraic concepts.

Number Sentence	Algebraic equation
_____ $+ 4 = 9$	$x + 4 = 9$
_____ $\times 3 = 15$	$3y = 15$
_____ $\div 2 = 8$	$\frac{m}{2} = 8$

4. Learners need to practice:
   - changing number sentences into algebraic equations
   - changing word problems into algebraic equations
   - describing an algebraic equation in their own words.
5. As with any area of mathematics, the more learners practise this concept the easier it will become.

Solving and completing equations

1. Learners need to be able to solve and complete equations using a combination of 2 methods:
   Inspection: which means solving by considering the information given and finding the solution through logic and a mental calculation.
   Trial and improvement: this means to try a value and check if it makes the statement true - in other words, does the left hand side equal the right hand side when the value chosen is used. If the first 'trial' doesn't work, choose another variable and try to improve on what was chosen before until the value chosen makes the left hand side equal to the right hand side.

Teaching Tip:
Learners need to always check to see if their answers make sense in the context of the question. This will be a good skill if applied later in the senior phase, when dealing with financial mathematics and even geometry.

2. Example: Solve $4x \neq 52$
   Learners would "guess" 12 (This is a trial which will need to be improved upon)
   $4 \times 12 = 48 \quad LHS = RHS$
   Learners now try 13 (This is an improvement as it gives an accurate result)
   $4 \times 13 = 52 \quad LHS = RHS$

Finding the value of an expression by means of substitution

1. Learners have been substituting values into rules as they worked with input and output diagrams, tables of values and even when calculating area and perimeter throughout the intermediate phase. This is merely the formalisation of their understanding.
2. Example: What number must be multiplied by 5 to get 45?
   Learners would by inspection determine the answer is 9.
   The check is done by substituting the 9 back into the equation and determining if the result is correct. Thus if x = 9 then 5 × 9 = 45
3. This can be extended to solving equations using formulae that learners are familiar with. This could include perimeter. For example: The length of a rectangle is 5m and the breadth (width) is 2m. What is the perimeter of the rectangle, if perimeter = 2 L + 2 B
   Perimeter = (2 × 5m) + (2 × 2m)
   Perimeter = 10m + 4m = 14m

Teaching Tip:
Learners will only need to solve equations that have natural number answers in this term but in Term 4 learner knowledge will be extended to include integer answers.

TOPIC 5: GRAPHS

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit covers a basic introduction to line graphs (Linear functions). In the intermediate phase learners only dealt with statistical graphs such as bar graphs and pie charts.
- It is important to ensure that learners can identify these graphs and their various characteristics as this topic becomes a larger component later in the FET phase.
- Although learners have been working with statistical graphs, there are a few marked differences in the structure and definitions used when dealing with these linear functions.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 6	GRADE 7	GRADE 8 and 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Bar graph interpretation
• Histogram interpretation
• Pie chart interpretation
• Concepts and skills developed in the intermediate phase relating to functions and relationships
	• Analyse and interpret global graphs of problem situations, with special focus on the following trends and features:
	  • linear or non-linear
	  • constant, increasing or decreasing
	• Draw global graphs from given descriptions of a problem situation, identifying features specific to that particular graph
		• Recognize and interpret linear and non-linear functions
		• Draw a wide variety of graphs using techniques related to those graphs
		• Sketching functions in contexts such as trigonometry and financial mathematics
		• Understanding trends in graphs with divergent data.
		• General graph theory

GLOSSARY OF TERMS

Term	Explanation / Diagram
Graph	A diagram showing the relationship between variable quantities
Variable	A quantity whose value can change
Dependant Variable	The quantity that is being observed
Independent Variable	A variable (often denoted by x) whose variation does not depend on that of another.
Linear Graph	A graph which is a straight line
Non-linear graph	A graph that does not form a straight line. It is usually in the form of a curve.
Increasing Function	As the x-values of the function increase, the y-values must increase as well. (Uphill from left to right)
Decreasing Function	As the x-values of the function increase, the y-value would decrease. (Downhill from left to right)
Constant Functions	A function whose value is the same for every input value. In other words, it is a straight line that is neither increasing or decreasing.

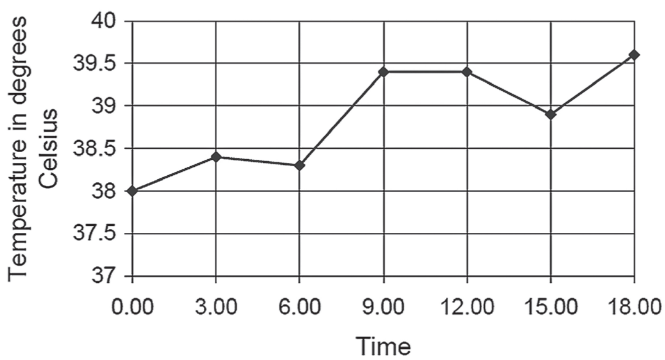
SUMMARY OF KEY CONCEPTS

Analyse and interpret global graphs

1. Learners have worked with statistical graphs in the intermediate phase. They are now going to investigate the trends and features of global graphs. Examples of global graphs include: the relationship between time and distance when travelling or the relationship between temperature and time of day.

Teaching Tip:
It may be a good idea to revisit the required skills relating to the drawing of axes and scale. Learners have practised these skills previously when drawing graphs in the earlier stages of the intermediate phase but may need reminding.

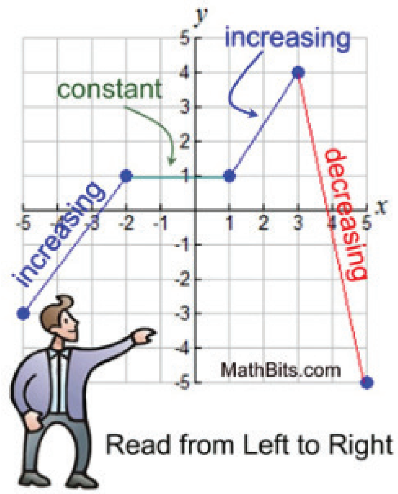
2. Example of a global graph:



Time needs to be spent discussing with learners what an entire graph means. These graphs tell stories and learners need to be encouraged to tell these stories themselves. Ask learners questions that make them look at the graph to interpret it and be able to answer the question.

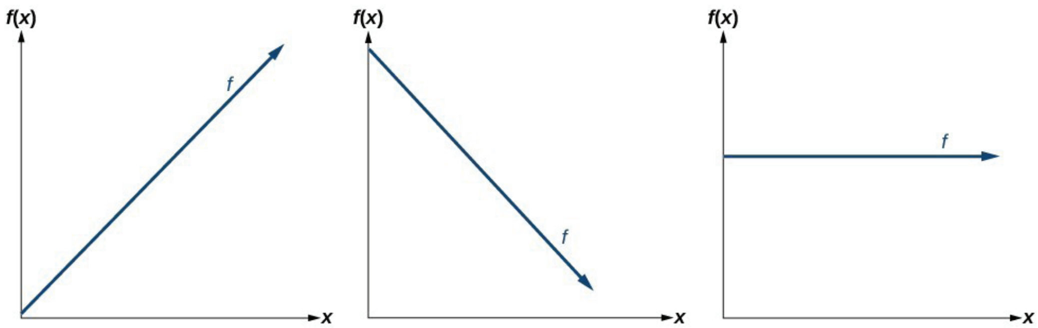
For example, What was the temperature at midnight? or During which two times was the temperature constant? These are just two of many possible examples.

3. Learners need to be able to identify a variety of trends including:



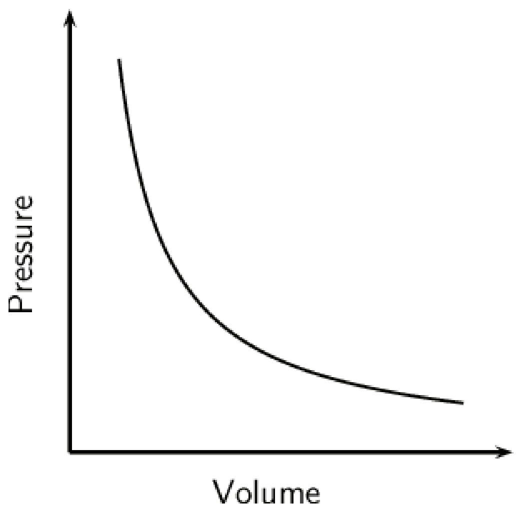
Linear Graph

Increasing function **Decreasing function** **Constant function**



(a) (b) (c)

Non-linear Graph



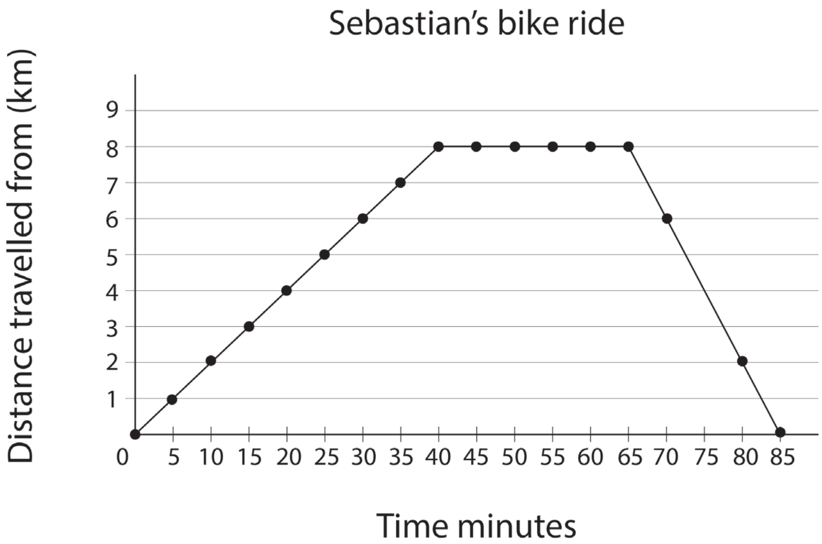
4. Learners need to understand that graphs tell a story. And that the trend describes the story visually.

5. Learners need to distinguish between the dependant and independent variable. If time is involved, it is ALWAYS represented on the horizontal axis as this represents the independent variable.

   For example, the number of sweets that can be bought depends on the cost (the cost would be independent and the number of sweets dependant).

   It is important that learners can distinguish between the dependant and the independent variable and there must be clarity on why they must be represented correctly on the relevant axes.

   This graph tells the story of a bike ride.



6. Learners need to be able to understand (and tell a story) from a graph such as this one and also be able to take information representing a situation and put it into a visual representation.

7. Some information which should be noted from this graph:
   - Sebastien starts his ride (0;0) – he has not yet travelled any distance (0m) and no time has yet elapsed (0s).
   - There is a constant speed from the time he starts riding for 40 minutes.
   - After 40 minutes he is 8km from home.

- From 40 minutes to 65 minutes (25 minutes in total) he doesn't travel any further. It seems he is having a rest.
- 65 minutes from when he first began his journey he sets off again back home.
- He arrives home after 90 minutes in total. In other words, the return part of the journey takes him 20 minutes (from the 70th minute to the 90th minute).

8. A discussion could be had as to why the return journey may have been quicker than the first part of his ride.

Drawing global graphs from a table of values

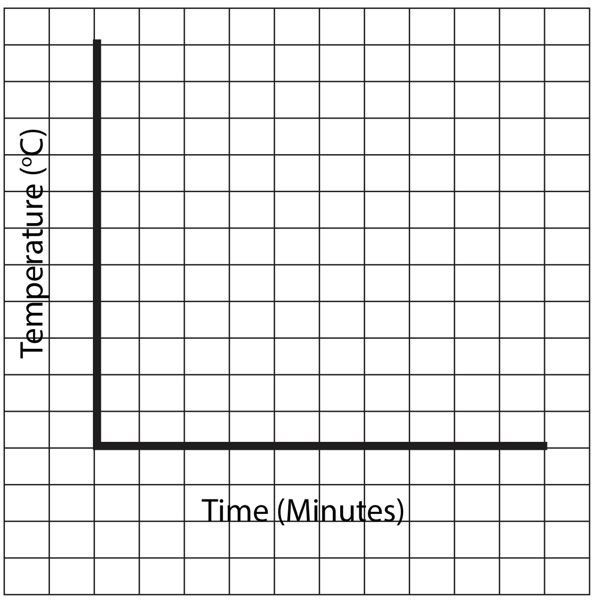
1. Drawing these global graphs from a table of given values is a important skill to ensure that learners can draw linear functions later in the senior phase.

Teaching Tip:
Learners will be using the skills acquired by drawing these graphs later in the senior and FET phase as they will be required to draw a variety of functions both linear and non-linear. Understanding the plotting of these points as global graphs will empower learners to plot the points from a table of values and doing so correctly.

2. Example:

The table below shows how quickly a beaker of water was heated up using a bunsen burner

Time (Minutes)	Temperature (°C)
0	10
1	20
2	30
3	40
4	50
5	60
6	70
7	80
8	80
9	90
10	100



TOPIC 6: TRANSFORMATION GEOMETRY

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the content area, 'Space and Shape (Geometry)' and counts for 25 % in the final exam.
- The unit covers transformations of 2D shapes. This includes symmetry, translation, rotation, reflections and enlargements or reductions of a variety of 2D shapes. Learners need to not only identify these transformations, but should be able to draw them on square paper.
- It is important to ensure that learners have a good understanding of these concepts as there will be reflections of Functions later in the FET phase and although more complex, the foundation is found in these transformations covered in Grade 7.
- Although learners have looked at lines of symmetry and some basic concepts of transformation in the intermediate phase, this section is covered extensively in Grade 7.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 6	GRADE 7	GRADE 8 and 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describing lines of symmetry in 2D and 3D objects found in nature. • Describing rotational symmetry of shapes • Enlargement/reductions of basic shapes (quadrilaterals and triangles only)	• Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size	• Transformations performed on a variety of shapes including 3D shapes. • Multiple transformations performed on a single shape • Glide reflections performed on shapes • Transformations performed on functions (graphs).

GLOSSARY OF TERMS

Term	Explanation / Diagram
Transformation	A change in the position, orientation or size of an object or shape
Rigid Transformation	Transformations in which size and shape are preserved (that means they do not change)
Translation	A transformation in which a shape moves left, right, up or down
Object	The original shape before a transformation has been made
Image	The resulting shape after a transformation has been made
Reflection	A mirror image of a shape that has been reflected
Line Of Symmetry	This is the line that divides the object exactly in half where the result is that one half is an identical mirror image of the other half
Vertical Symmetry	A vertical axis divides the shape in half
Horizontal Symmetry	A horizontal axis divides the shape in half
Radial Symmetry	Lines of symmetry can be drawn in all directions
Rotation	A transformation in which a shape is rotated or turned to give the resulting image
Point Of Rotation	The point around which a shape is rotated to deliver a desired image
Order Of Rotational Symmetry	The number of positions a shape can be rotated where the resulting image is identical to the original object
Enlargement	A transformation where the shape of the object is maintained but the size is increased.
Reduction	A transformation where the shape of the object is maintained but the size is decreased.
Centre Of Enlargement / Reduction	The point from which an enlargement takes place or the point towards which a reduction occurs
Scale Factor	The number of times the image is larger or smaller than the original object
Tessellation	A pattern of congruent (identical) shapes and images where there would be no gaps or overlaps
Congruent	Identical

SUMMARY OF KEY CONCEPTS

Transformations

1. All transformations in Grade 7 need to be done on squared paper.
2. As learners need to understand the various transformations really well by the end of Grade 7, it is important that each component is discussed in detail and that learners have mastered the separate skills before they start performing multiple transformations.
3. Learners need to understand very clearly the differences between dynamic transformations where the size of the shape is altered and rigid transformations where size is not affected and the image remains congruent to the original object

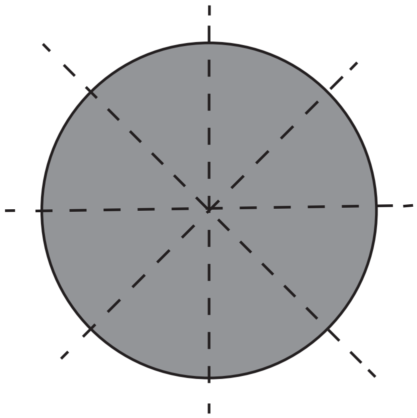
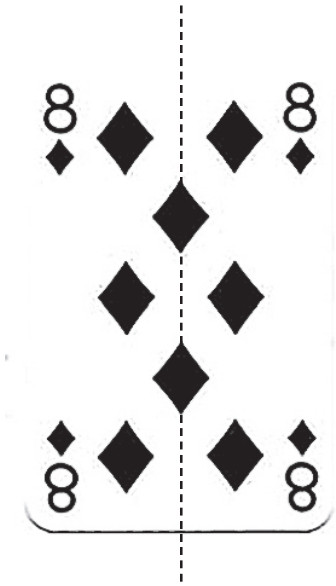
Lines of symmetry

1. Learners need to be able to identify and draw lines of symmetry in various shapes and objects. Learners need to distinguish between horizontal, vertical and radial lines of symmetry.

Teaching Tip:
Throughout the intermediate phase learners have identified and drawn lines of symmetry. Learners in Grade 7 must be able to identify these with ease and draw them accurately on squared paper.

Teaching Tip:
Learners can get a better understanding by drawing pictures on a page folded in half. This ensures that they recreate the same picture on the other half. Or they can cut paper that has been folded in half to have mirror image cut-outs where the fold line is an axis of symmetry. This is a practical component in the curriculum so learners can learn by experience.

2. Examples of symmetry

3. Rotational symmetry is also an important concept as learners need to understand that shapes may have as many as an infinite number of symmetry lines. It must be clearly defined that the order of rotational symmetry refers to the number of positions a shape can be rotated to still result in the original shape.

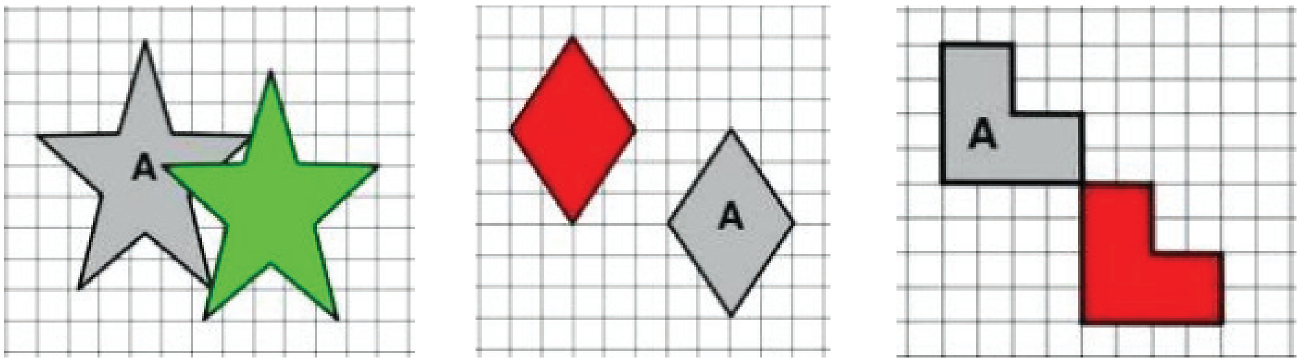
   Examples:
   The card below has an order of rotational symmetry of 2. In other words the card can turn until it looks identical to the original position. It then turns a second time to be back in the original position.

Translations

1. A translation moves an object from one place to another. This transformation must be performed on squared paper. Learners need to be able to identify the translation performed and they must also be able to draw translations from a set of instructions.

Teaching Tip:
Learners need to use the correct terminology when translating shapes, they must state if the shape is moved up or down, left or right and the number of units the shape is translated. Encourage learners to be clear when describing their translations.

2. Every point of the 2D shape must be moved by a fixed distance in a given direction.
3. Learners need to be able to translate a variety of unique shapes and not just shapes such as quadrilaterals and triangles. The shapes can have a multitude of sides and can be concave or convex.
4. Shapes must be translated multiple ways and not just a single directional movement at a time.
5. Example:

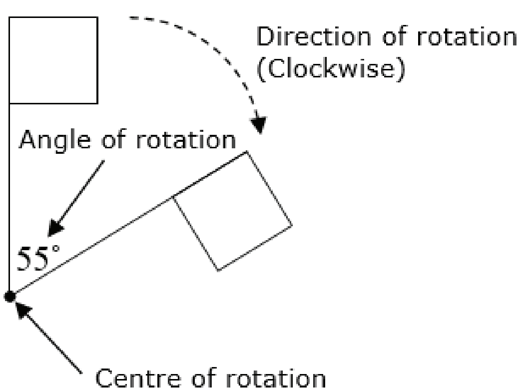


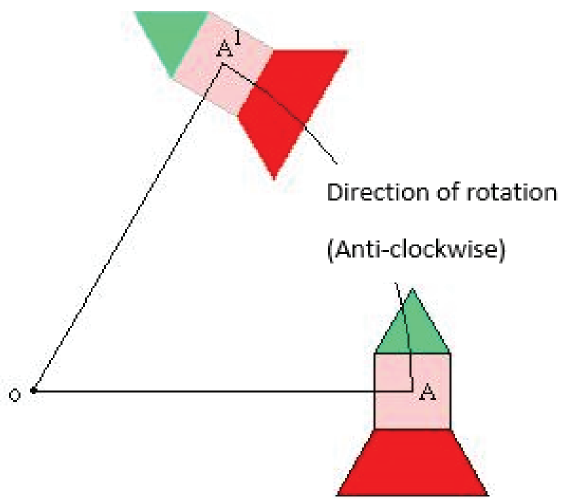
Rotations

1. A rotation turns a shape without changing the size of the shape and the shape is rotated around a point of rotation. This transformation must be performed on squared paper.

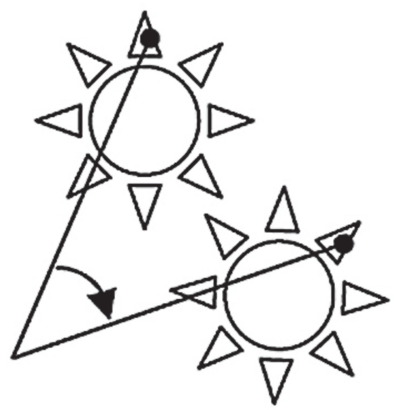
Teaching Tip:
Learners need to understand that the point of rotation affects the rotation of the shape. A good way to demonstrate this is to show learners how a shape rotates if the one corner is attached by a drawing pin and then the shape is rotated about that point. This is a great visual activity and helps the learners to understand the purpose of the rotational point.

2. This also links back to rotational symmetry as discussed earlier in this topic. Links must be made so that learners can understand the relationship between rotation and rotational symmetry.
3. Learners need to be made aware of the angles formed when objects are rotated.
4. They also need to be able to identify the rotational direct clockwise or anti-clockwise and they should start to see angles such as 90 degree angles and 180 degree angles
5. Example:

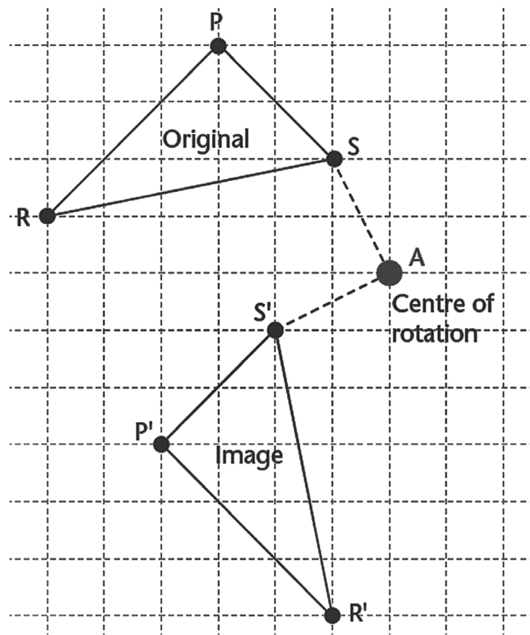




Learners need to also be capable of rotations around a fixed point of rotation.



The vertex of the angle created would be the rotational point (centre of rotation).

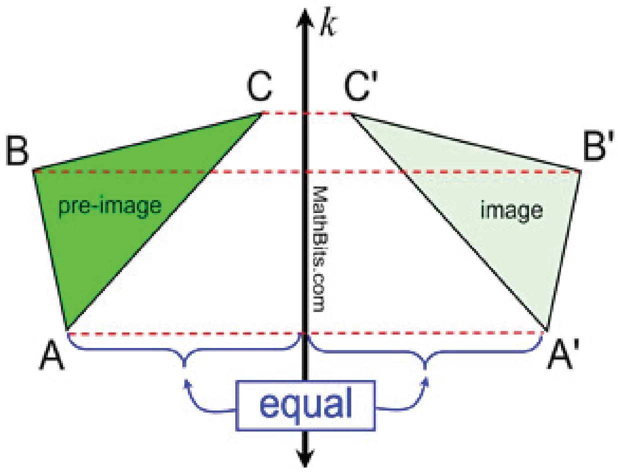


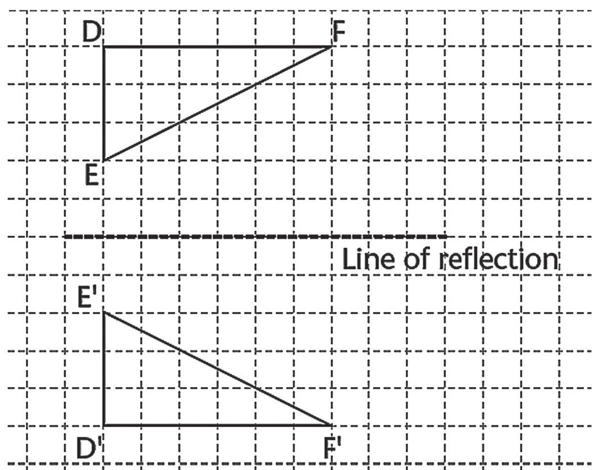
Reflections

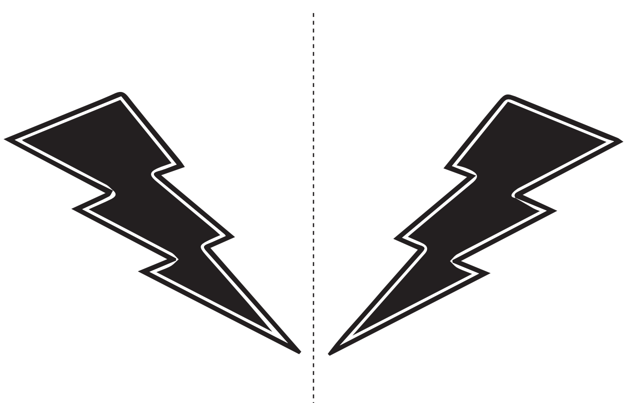
1. A reflection is a mirror image of a shape that occurs over an axis that is either vertical or horizontal in Grade 7. This transformation must be performed on squared paper.

Teaching Tip:
The best way to have learners understand a reflection is to use a mirror and have learners place it on a reflection line and then draw what they see in the other side of the line. This is a good visual technique especially because many learners have spatial problems and battle to understand the concept of reflections. If a mirror is not available folding the page on the line of reflection is also helpful.

2. A reflection line is the same as an axis of symmetry as the reflection of the object is identical on the other side of the reflection line.
3. Learners need to be able to identify a reflection as well as draw a reflection.
4. Example:







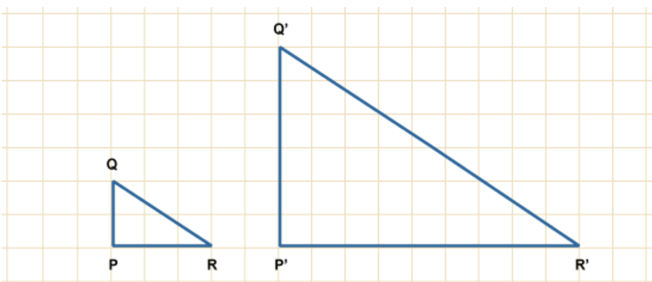
Enlargements and reductions

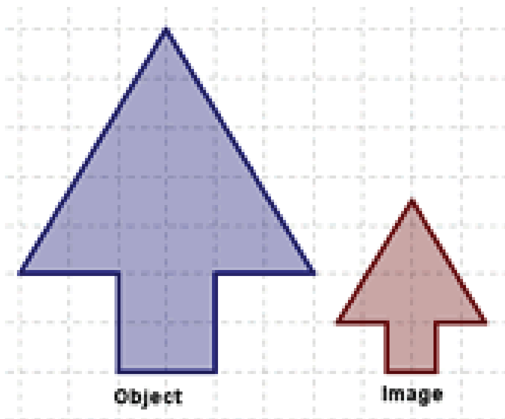
1. This transformation changes the size of the object and therefore the image is similar to the object and is no longer congruent. This transformation must be performed on squared paper.

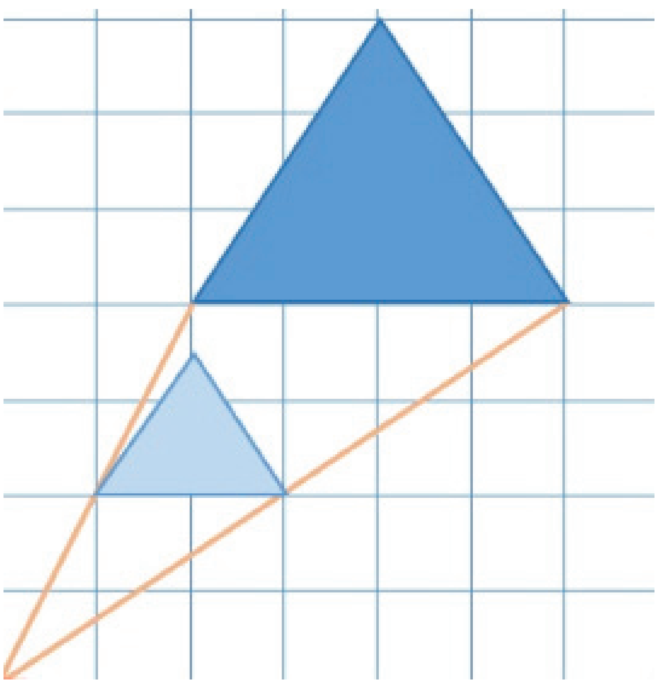
Teaching Tip:

A good understanding of scale factor will be useful when scaled drawings must be done later in the senior phase. Learners need to understand that scale factors larger than one result in enlargements and scale factors between zero and one result in reduced images. It is therefore important that educators explain that scale factor is always multiplied by the original size to result in the image. It is better to say the image is half the size of the original object as this makes it relatable to learners when the scale factor is times by $\frac{1}{2}$.

2. Learners are expected to enlarge and reduce varied shapes and not just triangles and quadrilaterals.
3. Example:

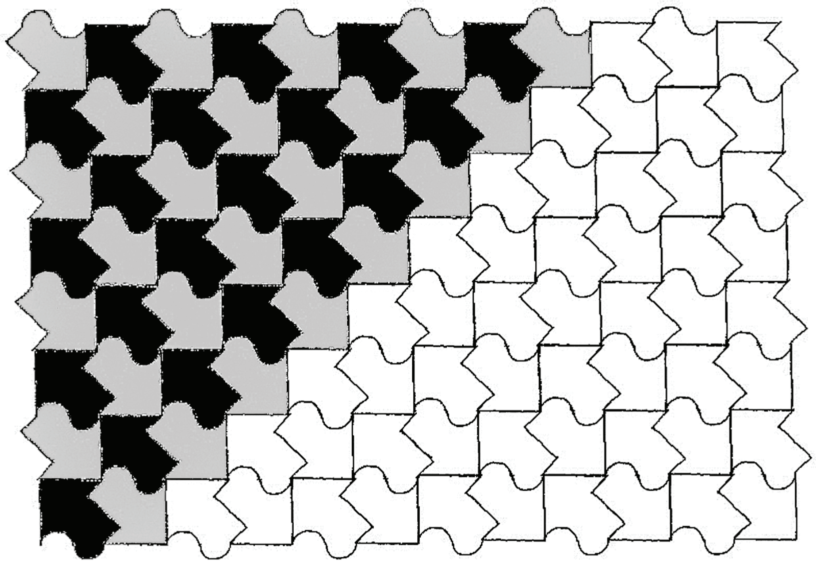






Tessellations

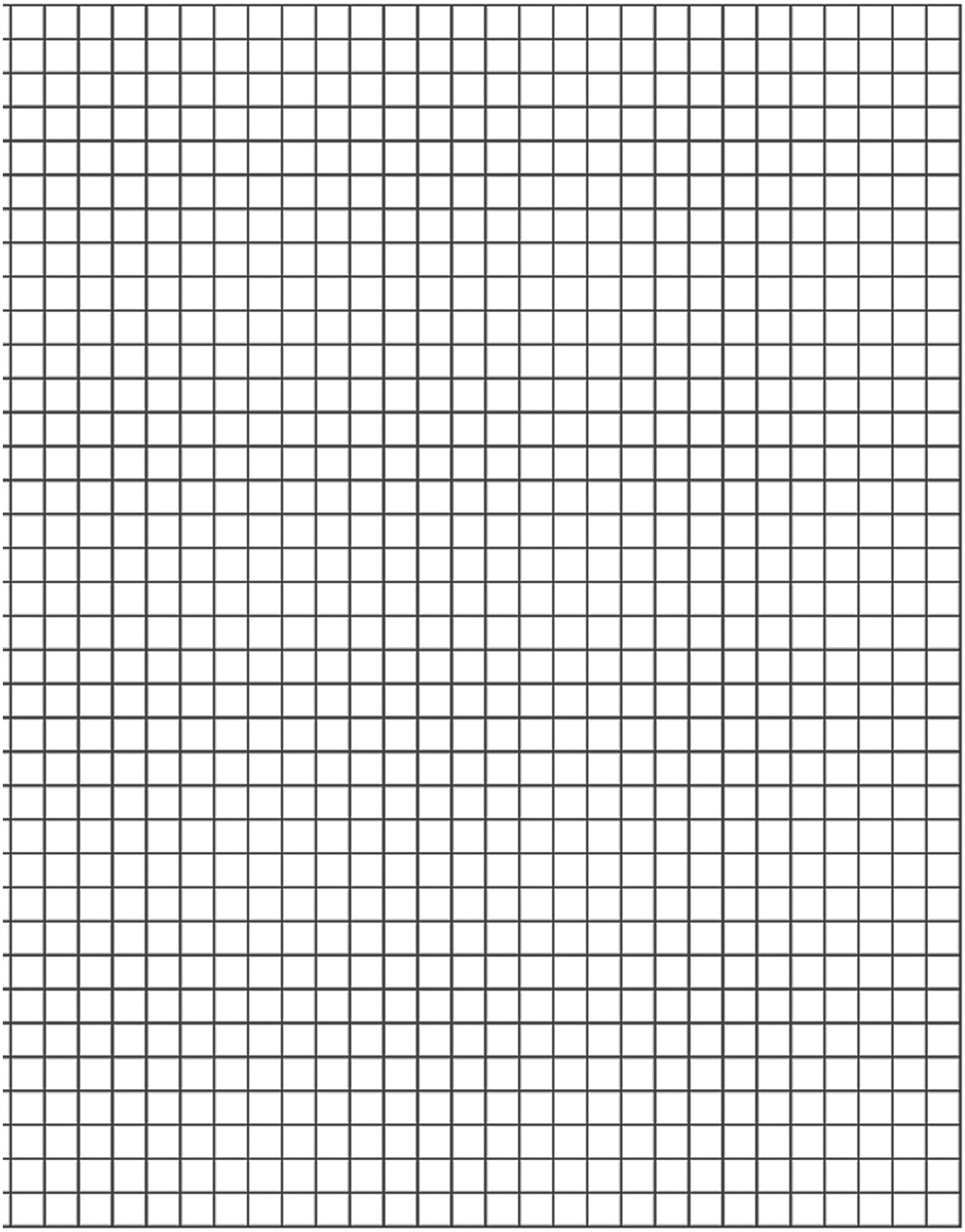
1. A tessellation is a pattern formed from identical shapes that fit perfectly together without any spaces between them. The shapes should not overlap
2. Learners need to be capable of basic tessellations where a variety of transformations are used to make the pattern tessellate. This is a good opportunity to assess a general understanding of the topic by asking learners to describe what transformations they have used in their tessellated pattern.



Tesselations can be fun to make and giving the learners a project to create their own is an excellent idea.

RESOURCES

Squared paper



TOPIC 7: GEOMETRY OF 3D OBJECTS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the content area, 'Space and Shape (Geometry)' and counts for 25 % in the final exam.
- The unit covers the recognition and classification of 3D shapes as well as the construction of 3D models by means of nets.
- This topic covers a visual understanding of 3D polyhedra which is expanded later in the senior phase and the FET phase where the emphasis is on the application of surface area and volume of 3D shapes.
- Although learners have previously looked at 3D shapes the focus in Grade 7 is the identification of the number of faces, vertices and edges as well as the construction of 3D models using nets of these shapes. Learners determine the final product based on the number and type of faces the shape has.
- This is a practical component of the syllabus so make it fun for the learners and focus on identification of actual objects as well as construction using materials that are easy for all learners to obtain and use.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 6	GRADE 7	GRADE 8 and 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describe, order and compare 3D objects in terms of faces, vertices and edges • Make 3D models using drinking straws, toothpicks • A basic introduction to the use of nets in construction of 3D models	• Describe, sort and compare polyhedra in terms of: shape and number of faces, number of vertices and number of edges • Revise using nets to create models of geometric solids, including: cubes and prisms	• Defining the internal angle size using Euler's formulae • Polyhedra and their classification as per faces, vertices and edges. • Volume and surface area of a variety of 3D shapes and combined shapes.

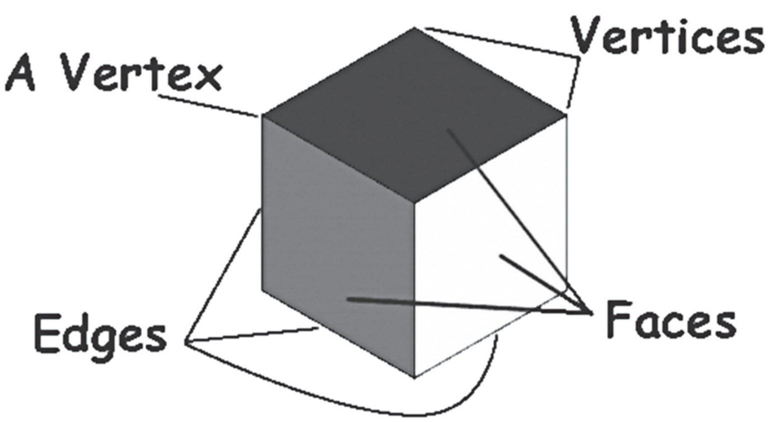
GLOSSARY OF TERMS

Term	Explanation / Diagram
Hexagon	A shape that has 6 sides
Regular Prism	A prism where the ends are regular polygons
Heptagon	A shape with 7 sides
Pentagon	A shape with 5 sides
Octagon	A shape with 8 sides
Plane Of Symmetry	A flat surface that cuts a 3D shape into two identical shapes
Net	A Flat "plan" that can be folded into a 3D shape with tabs that hold it together. It is a collection of polygons which can be folded up to make a polyhedron
Construct	To build something using a plan and suitable materials
Polyhedron/Polyhedra	A 3D shape where all the faces are polygons
Face	A flat surface of a shape
Edge	Faces meet forming a line that is an edge of the shape
Vertices	A point where two or more edges meet, also seen as a corner of the shape
Cube	A hexahedron with 6 square faces
Rectangular Prism/Cubiod	A hexahedron with 6 rectangular faces
Pyramid	A polyhedron that has any polygon as its base and then all other faces are triangles meeting at the apex (top of the polyhedron shaped in a point)
Cylinder	Two identical circles held together by a curved surface that is actually a rectangle.
Prism	A polyhedron with two identical faces held together by rectangles.

SUMMARY OF KEY CONCEPTS

Classifying 3D objects

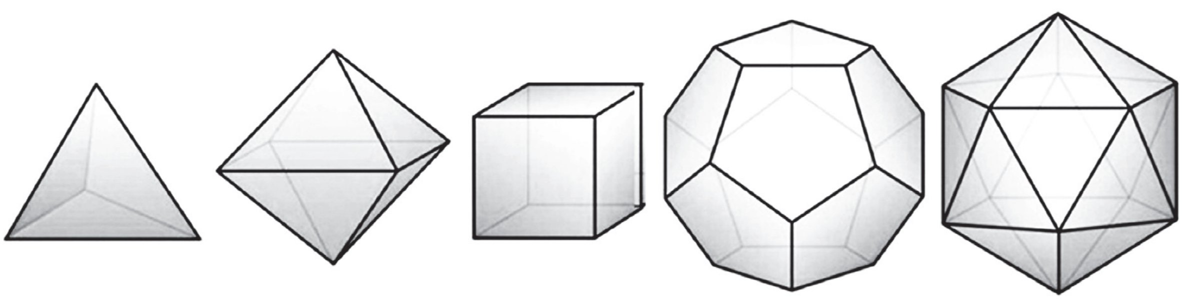
1. The focus of the classification of 3D objects is the classification by means of faces, edges and vertices.
2. Example:



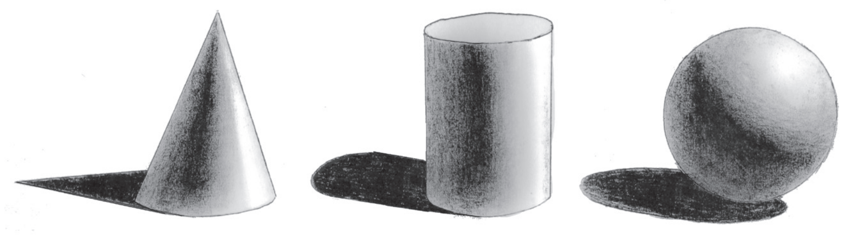
Teaching Tip:
Use as many manipulatives as possible (ask learners to bring 3D objects from home) and spend some time allowing the learners to hold the objects and count the faces, edges and vertices.

Teaching Tip:
Tell learners that they could draw a face on a face – this may help them remember where the faces are. (It isn't possible to draw on an edge or a vertex)

3. Learners need to know that most shapes are polyhedra but there are shapes such as cylinders that do not fall into this classification as the curved side joining the circular faces is not a flat surface.
4. Example: Polyhedron



Not polyhedron



5. Learners need to be able to name a given shape as well as describe the reason for the name they have chosen for the shape.

Prisms and Pyramids

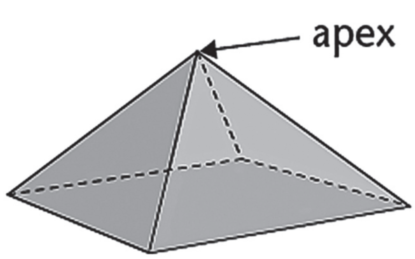
1. Learners need to be able to identify and distinguish between any prism (both regular and irregular) and a pyramid.

Note: Learners have already investigated the differences in Grade 6 so they should be aware of the basic structure that sets prisms apart from each other and apart from pyramids.

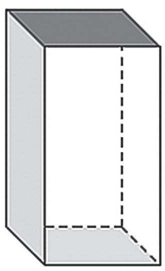
Teaching Tip:
Many learners find it easier to distinguish between shapes if they can touch the actual shape. It is a good idea for the teachers to construct a large cardboard model of the shapes. Students can touch and discover the differences and this helps them visualise the actual structure of the shapes they are learning about.

2. It is important to discuss the properties of prisms thoroughly and then discuss the properties of pyramids thoroughly as this will ensure a very clear understanding of each individual type of shape before learners are expected to tell them apart.

Pyramid

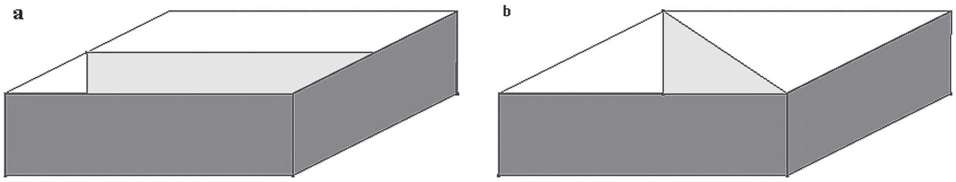


Prism



3. Planes of symmetry must be discussed and learners should be able to draw them into a given shape correctly thus they must have a clear understanding.

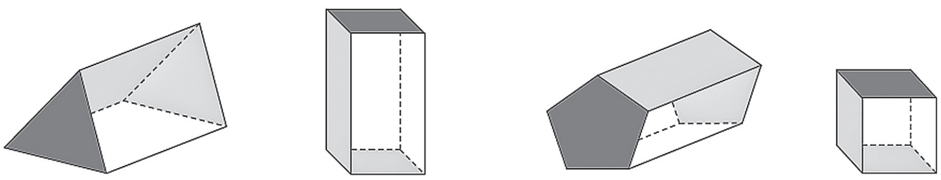
4. Examples:



The light grey plane divides the 3D shape into 2 equal parts above thus these are planes of symmetry.

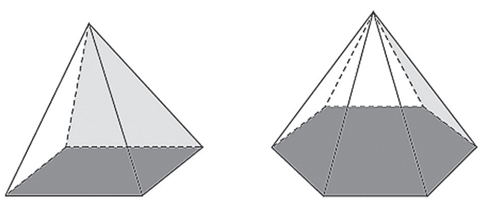
Teaching Tip:
The previous topic dealt with symmetry in 2D shapes. It may be a helpful exercise to revisit one of those examples if learners are struggling to grasp what a plane of symmetry should do.

5. Examples of prisms:



Triangular prism | *Rectangular prism* | *Pentagonal prism* | *Cube*

6. Examples of pyramids:

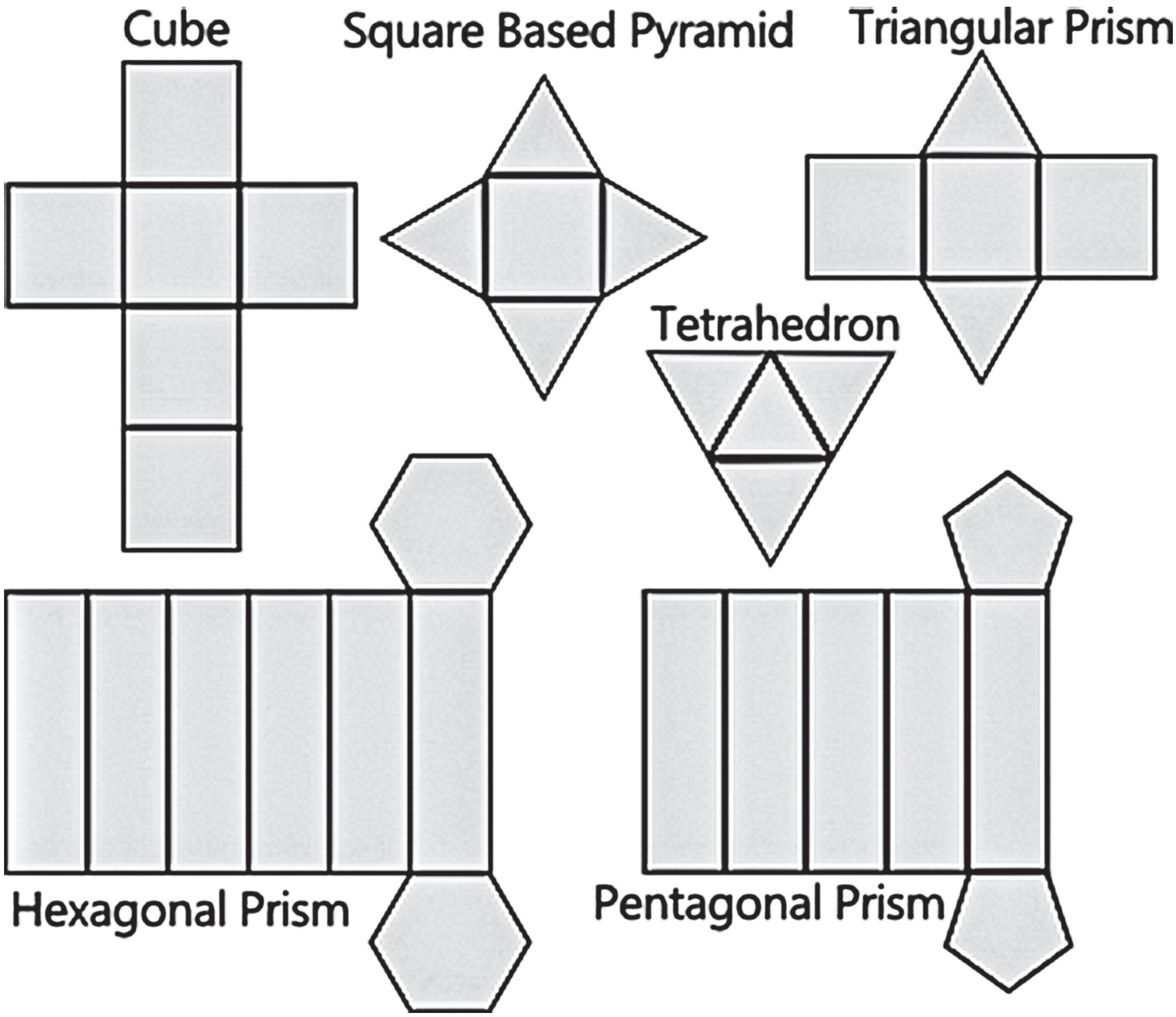


Square-based pyramid
It has 5 faces:
1 square,
4 triangles

Hexagonal-based pyramid
It has 7 faces:
1 hexagon,
6 triangles

Nets and their 3D solids

In order for a 3-dimensional shape to be made, a 2-dimensional net is required.
A net is the shape that needs to be cut out in order to fold up and become a 3-dimensional shape.
The following diagram shows the nets of some of the shapes dealt with in this topic:

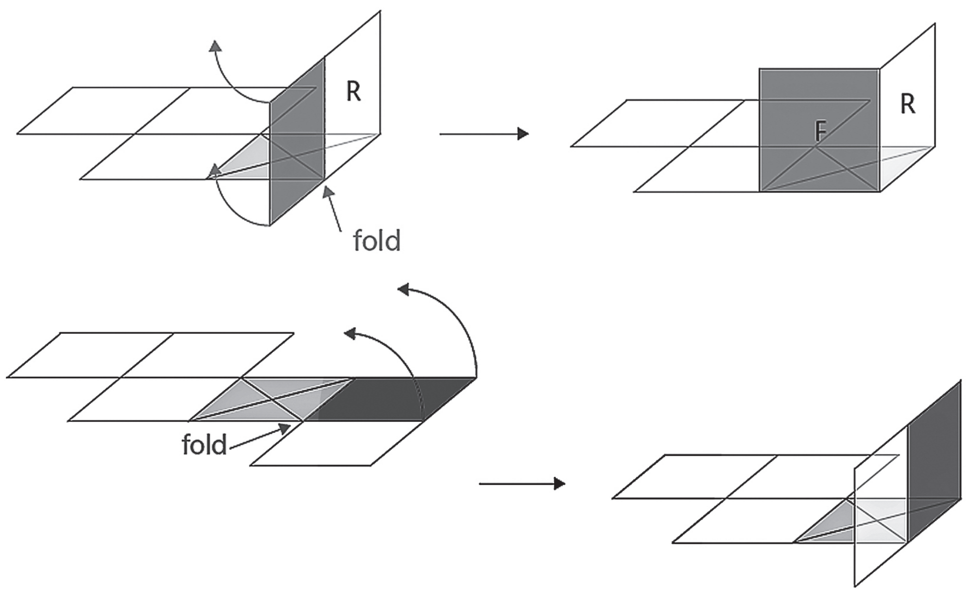


Learners need to be able to match a net with its 3D shape as well as make a net for a 3D shape.

Building 3D models

1. Learners are expected to produce models of 3D objects using nets.

Teaching Tip:
In Grade 6 learners were shown nets that result in shapes. Learners need to revisit the requirements for a good net, such as tabs, to ensure their success in constructing these 3D models. It will also help if learners are given clear instructions when they are constructing their shapes.
Example:



2. Learners need to not only be able to use a net, they should also be able to draw a net that would result in a desired shape.

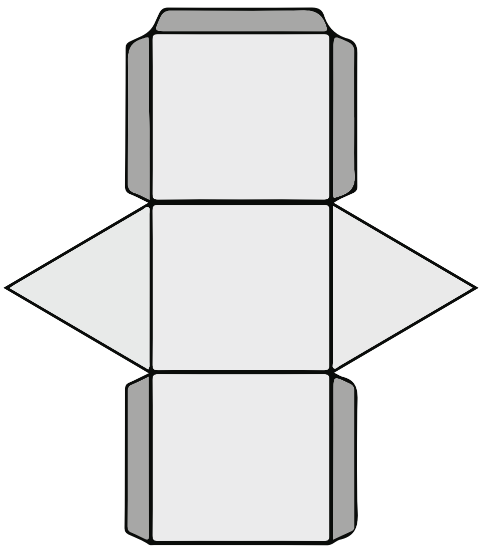
Teaching Tip:
Boxes used for packaging can be used to demonstrate shapes broken down into their nets. Smartie boxes, biscuit boxes, milk cartons, almost any object that comes packaged in cardboard is a good net to show learners how to visualize the net they wish to draw when given a shape.

There are a number of resources on the following pages that can be used in this section

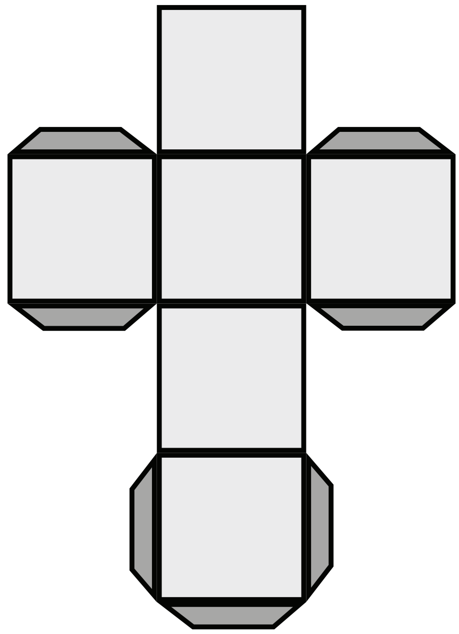
RESOURCES

NETS

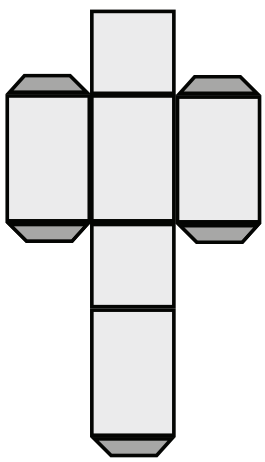
Triangular Prism



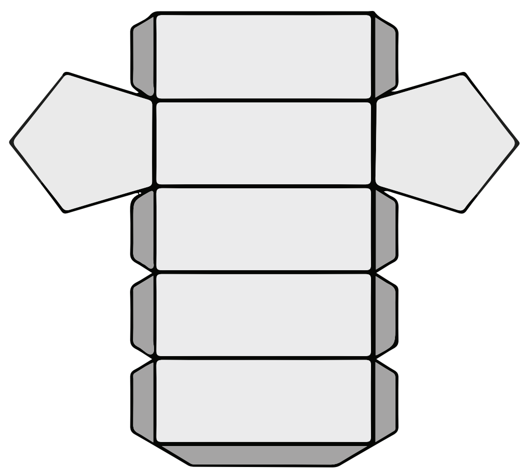
Cube

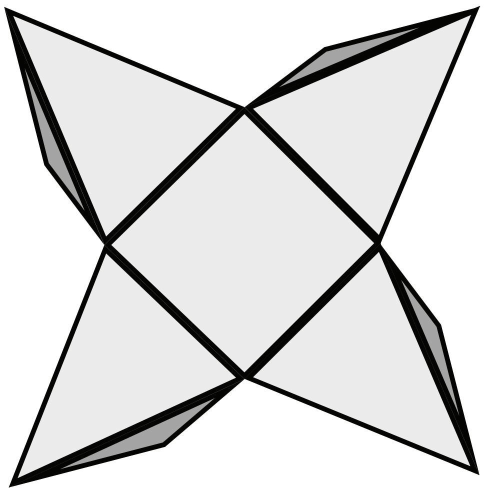
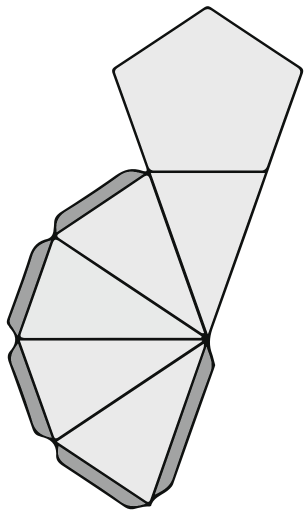
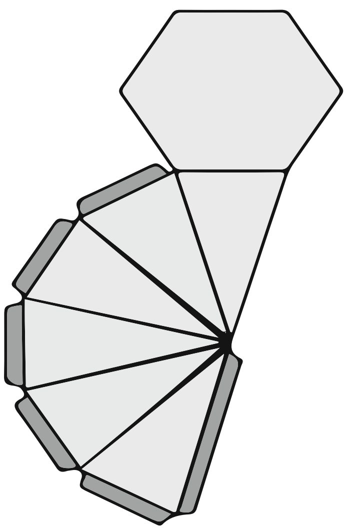
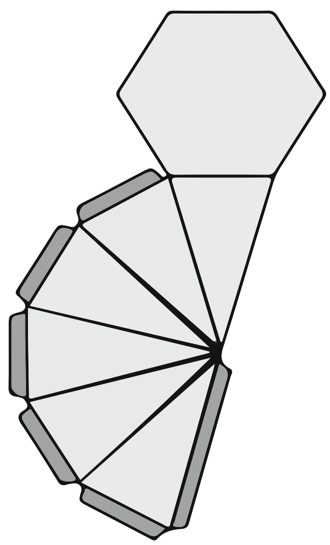


Rectangular Prism

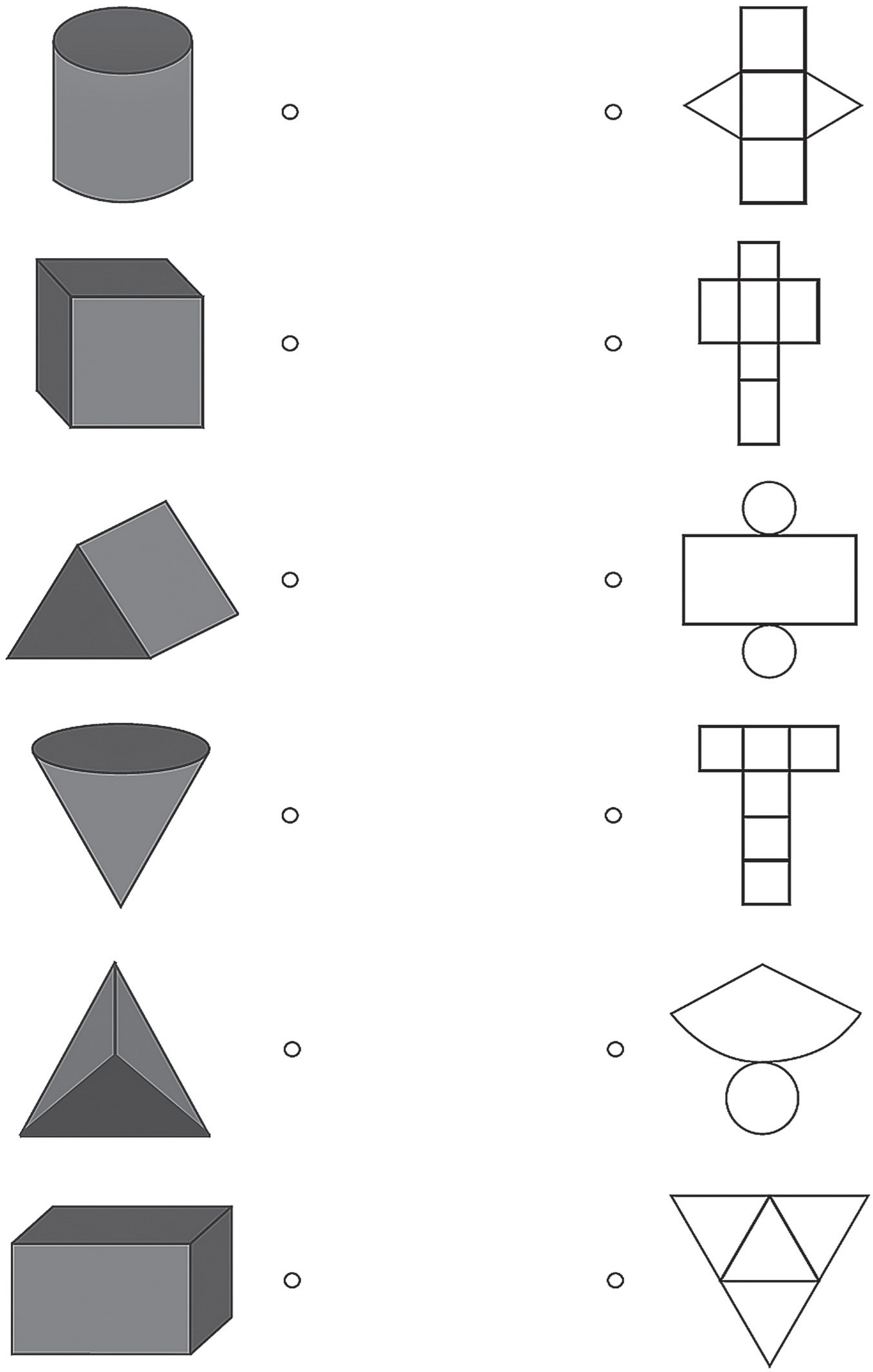


Pentagonal Prism

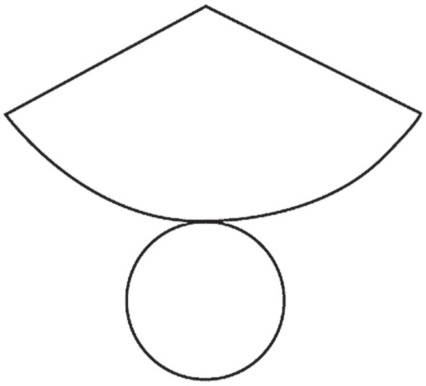


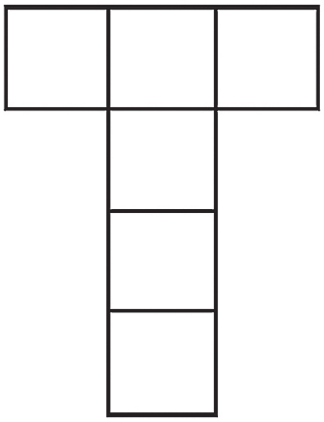
Square Base Pyramid 	Pentagonal Base Pyramid 
Hexagonal Base Pyramid 	Hexagonal Base Pyramid 

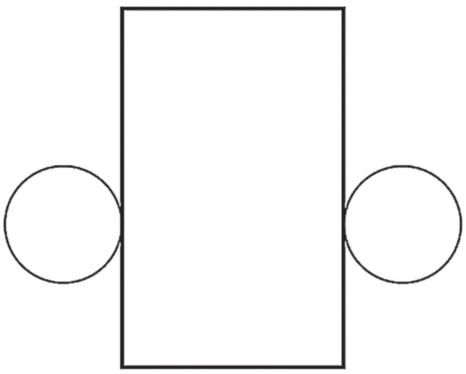
Join each shape to the matching net.

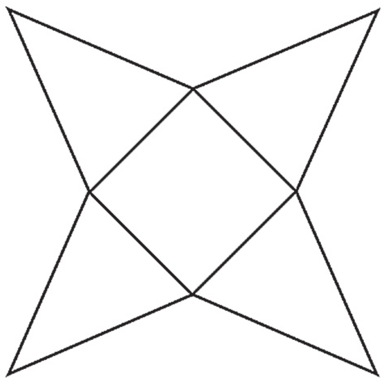


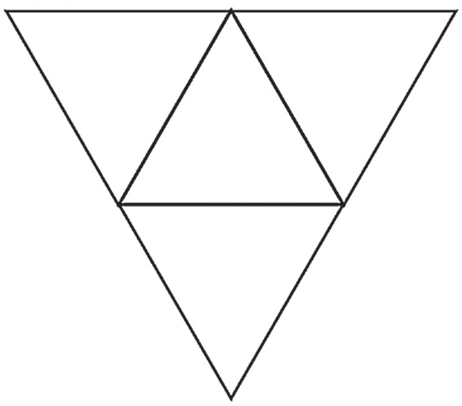
Name the solid shape that can be formed by each net.

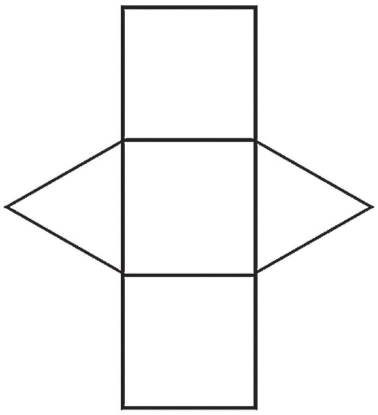












Notes

Notes